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Altitude for Maximum Angular Velocity About the Earth During Hypersonic Cruise

Nikos Markopoulos* and Kenneth D. Mease†

Princeton University, Princeton, New Jersey 08544

Nomenclature

C_A	= combined aerodynamic coefficient, $C_L + C_D \tan(\alpha - \varepsilon)$
C_D	= aerodynamic drag coefficient
C_L	= aerodynamic lift coefficient
g	= local gravitational acceleration
m	= mass of vehicle
r	= radius (altitude) of flight
S	= reference area
T	= engine thrust
t	= time
V	= flight speed
α	= angle of attack
γ	= flight-path angle
ε	= thrust direction angle relative to vehicle longitudinal axis
η	= throttle setting
μ	= gravitational constant
ρ	= atmospheric density
φ	= range angle (true anomaly)

Introduction

FOR hypersonic cruising flight there exists an altitude resulting in maximum angular velocity about a nonrotating planet. Although such an altitude does not correspond to minimum fuel consumption, it would be the ideal one at which to fly to minimize the time of flight between two given points on a planet's surface (neglecting of course the times of ascent and descent). Using the theory of ordinary maxima and minima, one can identify this altitude and the corresponding value of the maximum angular velocity by means of a very simple graphical solution. The authors stumbled upon this result while working on the longitudinal stability of hypersonic cruising flight,[1] and since they have not found it documented in the literature, they present it here as a short Note with the hope that it will be of some practical or at least of some academic or theoretical interest.

Equations of Motion

Consider the longitudinal translational motion of a high-performance aircraft, in the plane of a great circle, over a nonrotating spherical Earth. The aerodynamic plus propulsive forces acting on the vehicle during such motion can be accounted for by their components F_T and F_N that are, respectively, tangent and normal to the flight path. Explicitly,

$$F_T = T\cos(\alpha - \varepsilon) - (C_D/2)\rho S V^2 \tag{1}$$

$$F_N = T\sin(\alpha - \varepsilon) + (C_L/2)\rho S V^2 \tag{2}$$

It will prove useful to define a combined aerodynamic coefficient C_A by

$$C_A = C_L + C_D \tan(\alpha - \varepsilon) \tag{3}$$

Then, using C_A, F_N can be related to F_T via

$$F_N = F_T \tan(\alpha - \varepsilon) + (C_A/2)\rho S V^2 \tag{4}$$

Under the assumption of constant mass, the differential equations governing the motion of the center of mass of the vehicle can be written as

$$\dot{V} = (F_T/m) - g\sin\gamma \tag{5}$$

$$\dot{\gamma} = (F_N/mV) - [g - (V^2/r)](\cos\gamma/V) \tag{6}$$

$$\dot{r} = V\sin\gamma, \qquad \dot{\varphi} = V\cos\gamma/r \tag{7}$$

where the dot denotes differentiation with respect to t. It will be assumed in the remainder of this Note that 1) the flight regime is hypersonic, that is, V is very large; 2) the atmospheric density is a function only of altitude (radial distance), i.e., $\rho = \rho(r)$; 3) the local gravitational acceleration is inverse square, i.e., $g = \mu/r^2$; and 4) the thrust is an arbitrary function of the throttle setting, speed, altitude, and angle of attack of the vehicle, namely, $T = T(\eta, V, r, \alpha)$. In general, the aerodynamic lift and drag coefficients are functions of the Reynolds number, Mach number, and angle of attack of the vehicle. Because of the assumption of hypersonic regime, however, in the remainder of this Note the dependence of these coefficients on the Reynolds number and Mach number will be completely neglected.[2]

Equilibrium Solutions

Cruising flight, which will be denoted using a zero subscript, corresponds to an equilibrium solution of the fourth-order system, Eqs. (5–7), and is obtained by determining constant values of the states and controls that satisfy these differential equations. Clearly, the cruising flight path must be a great circle. Once its constant radius r_0 (corresponding to the cruise altitude) and the constant angle of attack α_0 along it are specified, the remaining steady states and controls are given by $\gamma_0 = 0$, $\varphi_0 = V_0/r_0$, and

$$V_0 = \left[\frac{2\mu m}{2mr_0 + SC_{A0}\rho_0 r_0^2}\right]^{\frac{1}{2}} \tag{8}$$

$$T_0 = T(\eta_0, V_0, r_0, \alpha_0) = \frac{C_{D0}\rho_0 S V_0^2}{2\cos(\alpha_0 - \varepsilon_0)} \tag{9}$$

$$C_{A0} = C_{L0} + C_{D0}\tan(\alpha_0 - \varepsilon_0) \tag{10}$$

Note that it is assumed here that for given r_0, α_0, and ε_0, Eq. (9) has one and only one solution for the nominal value η_0 of the throttle

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*Graduate Research Assistant, Department of Mechanical and Aerospace Engineering; currently Postdoctoral Fellow, Department of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150. Member AIAA.

†Assistant Professor, Department of Mechanical and Aerospace Engineering; currently Associate Professor, Department of Mechanical and Aerospace Engineering, University of California, Irvine, Irvine, CA 92717-3975. Associate Fellow AIAA.

setting. Note also that the value of α_0 cannot be specified freely but is subject to constraints related to the rotational dynamics of the vehicle. In particular, as the steady flight altitude increases and the atmospheric density goes to zero, the aerodynamic pitching controls become ineffective and the only equilibrium values for α_0 are the solutions to the equation $\sin 2\alpha_0 = 0$.

Cruise Altitude for Maximum Angular Velocity About a Planet

At the cruise condition F_T is zero, and from Eq. (4) the corresponding component F_N is proportional to, and has the same sign as, the combined aerodynamic coefficient C_A:

$$F_{N0} = (C_{A0}/2)\rho_0 S_0 V_0^2 \tag{11}$$

Positive C_{A0} thus means that the centripetal acceleration required to turn the vehicle along the great circle is provided by the difference between the (local) weight of the vehicle and (the sum of) the propulsive plus aerodynamic forces. The unrealistic case of negative C_{A0} on the other hand implies that propulsion plus aerodynamics cooperate with gravity in supplying this centripetal acceleration. It will now be shown that, for a given vehicle, and for the conventional case of positive C_{A0}, there exists an altitude at which the cruising flight speed given by Eq. (8) results in the maximum possible angular velocity Ω_0 about the Earth. Since $\Omega_0 = V_0/r_0$, from Eq. (8) one obtains explicitly

$$\Omega_0 = \left[\frac{2\mu m}{2mr_0^3 + SC_{A0}\rho_0 r_0^4}\right]^{\frac{1}{2}} \tag{12}$$

The quantities μ, m, S, and C_{A0} are independent of r_0. Thus, the maximum value of Ω_0 is reached at the altitude for which the denominator of the expression within the bracket in Eq. (12) becomes a minimum. Defining the dimensionless first-order atmospheric density gradient by

$$\sigma_1 = \frac{r_0}{\rho_0}\left(\frac{d\rho}{dr}\right)_0 \tag{13}$$

and setting the derivative of this denominator with respect to r_0 equal to zero, one obtains, in kilograms per square meters,

$$\frac{m}{SC_{A0}} = -\frac{(4+\sigma_1)r_0\rho_0}{6} \tag{14}$$

For the Earth's atmosphere, because of the sharp decrease of atmospheric density with altitude, the right-hand side of Eq. (14) is monotonically decreasing with r_0. Thus, for positive C_{A0} only, Eq. (14) provides no more than one solution for r_0, for which the corresponding value of Ω_0 is given, in degrees per hour, by

$$(\Omega_0)_{\max} = \frac{648{,}000\mu^{\frac{1}{2}}}{\pi r_0^{\frac{3}{2}}}\left(\frac{4+\sigma_1}{1+\sigma_1}\right)^{\frac{1}{2}} \tag{15}$$

Since the left-hand side of Eq. (14) is a characteristic of the vehicle, whereas the right-hand sides of Eqs. (14) and (15) are characteristics of the atmosphere, the following graphical solution to the problem is suggested: one needs to construct only once a (double) plot of the right-hand sides of Eqs. (14) and (15) vs altitude. Then, for any vehicle, one can calculate the left-hand side of Eq. (14) and then read from this plot the altitude satisfying Eq. (14) and the corresponding maximum value of Ω_0. That this value is indeed a maximum can be argued by showing that the corresponding second derivative of Ω_0 with respect to r_0 is strictly negative. The variations of the negative of σ_1 and of ρ_0 with altitude are produced[3,4] in Fig. 1, from which it can be seen that in the altitude range of interest the magnitude of σ_1 is very large and the term within the square root on the right-hand side of Eq. (15) is very nearly equal to unity. This leads immediately to the conclusion that the maximum value of Ω_0 is approximately equal to its circular orbital value at the best r_0, in degrees per hour,

$$(\Omega_0)_{\max} \approx (\Omega_0)_{\text{orb}} = \frac{648{,}000\mu^{\frac{1}{2}}}{\pi r_0^{\frac{3}{2}}} \tag{16}$$

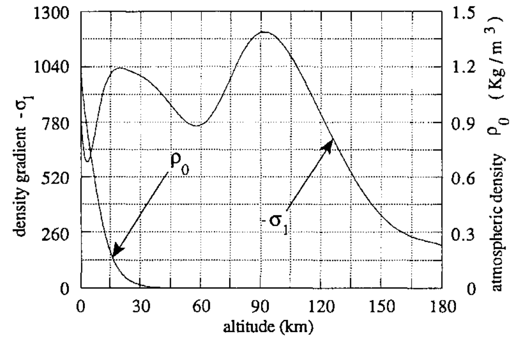

Fig. 1 Variation with altitude of atmospheric density and of first-order, dimensionless, atmospheric density gradient.

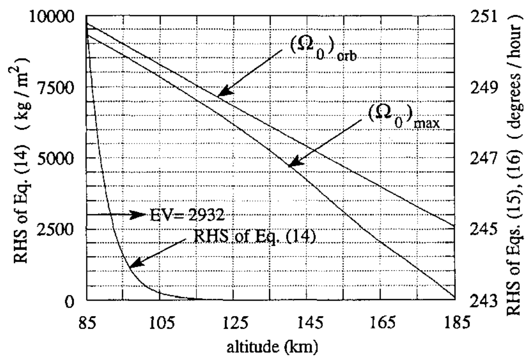

Fig. 2 Right-hand sides of Eqs. (14–16) vs altitude.

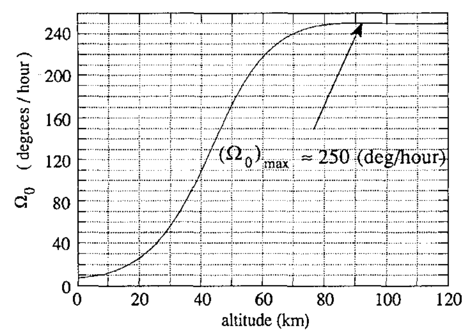

Fig. 3 Cruising flight angular velocity about the Earth vs altitude for Etkin's[5] vehicle.

Figure 2 shows the actual plots vs the altitude of the right-hand sides of Eqs. (14–16) and confirms the preceding conclusion. As an example, for a particular set of vehicle parameters used previously by Etkin[5] ($C_{L0} = 0.05$, $C_{D0} = 0.0133$, $m/S = 146.6$ kg/m^2, $\alpha_0 = 0$, and $\varepsilon_0 = 0$), the left-hand side of Eq. (14) is approximately equal to 2932 kg/m^2. Entering this number into Fig. 2, one finds that the maximum possible value of Ω_0 is approximately equal to 250 deg/h and can be achieved by flying at an altitude of 92 km, a result that is verified by plotting Ω_0 [Eq. (12)] vs the altitude in Fig. 3. Note that during an actual trip the mass of the vehicle slowly decreases, so that for a given C_{A0} this best altitude increases and the maximum value of Ω_0 decreases.

Conclusion

An altitude has been identified that for hypersonic cruising flight results in maximum angular velocity about a nonrotating planet. This altitude depends on the wing loading of a vehicle and on its combined aerodynamic coefficient. Increasing the former or decreasing the latter lowers the value of this altitude and raises the value of the corresponding maximum angular velocity. The results obtained here are valid in the absence of any constraints. In practice, cruising flight deep within the atmosphere at very high values of the angular velocity may not be possible because of constraints related to the maximum allowable aerodynamic heating rate, dynamic pressure, etc. In such a case the results presented here can be modified to identify an altitude resulting in the maximum angular velocity about a planet while at the same time satisfying all of the constraints.

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Periodic Motion in the Tethered Satellite System

Shaohua Yu*

Chinese Academy of Sciences, Beijing 100080, People's Republic of China

Introduction

THE tethered satellite system (TSS) has been proposed and investigated for many years. Colombo et al.[1] proposed a shuttle-borne tethered subsatellite and demonstrated the gravity gradient stability of the system. The subsatellite may be deployed upward or downward to perform various scientific experiments on either electrically conducting or nonconducting tethers.[2,3] The TSS-1, a NASA/Agenzia Spaziale Italiana (ASI) joint project, was flown in 1992 to verify the electrodynamic tether and system technology.[4]

The TSS has the deployment (DE), station-keeping (ST), and retrieval (RE) phases of motion of the subsatellite. Rupp[5] proposed a tether tension control law and greatly advanced the understanding of the mathematic model, dynamics, and control of the system. A group of specialists from ASI, NASA, and Martin Marietta adopted a pragmatic approach to the system control,[6] which consisted of the following: 1) designing a mission profile for the tether length variation; 2) computing step by step the other variables in the system (for example, the tether tension), based on a mathematical model and the mission profile; and 3) implementing a tether tension control in accordance with the computed results.

Although this control was not closed loop, it was very useful. In Refs. 7 and 8 the so-called tether length rate control algorithm (LRCA) was adopted, by which the system control was transformed into the following: 1) selecting the desired values for the parameters of LRCA; 2) predicting the system's behavior under the selected parameters, based on the nonlinear dynamic system analysis; and 3) implementing LRCA in real time for the system control.

LRCA worked very well in all of the three phases of TSS motion. The controlled subsatellite's trajectory by LRCA was stable and straight line in DE and RE phases and a stationary point in the ST phase.

However, the results obtained so far are restrained by an assumption of a circle orbit of the mother satellite in the system. In the case of an elliptic orbit, the system enters a periodic motion. This Note addresses a mathematical model of the system, the length rate control algorithm, a numerical method for computing the periodic motion, the stability and domain of attraction of the periodic motion, and also a computer-simulated trajectory of the subsatellite.

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*Professor, Control System Dynamics, Center for Space Science and Applied Research, P.O. Box 8701.

Mathematical Model and Control Algorithm

Under the commonly accepted assumptions that the two satellites of TSS are coplanar, the equations of motion of the subsatellite are given in the following form[7,8]:

$$\ddot{D} - D(\dot{\upsilon} + \dot{\varphi})^2 + (\mu/RM^3)D(1 - 3\sin^2\varphi) = -(T/ms) \quad (1)$$

$$D(\ddot{\upsilon} + \ddot{\varphi}) + 2\dot{D}(\dot{\upsilon} + \dot{\varphi}) - 3(\mu/RM^3)D\sin\varphi\cos\varphi = 0 \quad (2)$$

where μ is the gravitational coefficient of the Earth, RM is the orbital radius of the mother satellite, υ is the true anomaly on the orbit, T is the tether tension, and ms is the mass of the subsatellite. Equation (1) describes basically the variation of the distance D between the two satellites (tether length), whereas Eq. (2) describes the variation of the direction angle φ of the tether line. The term φ is measured from the local horizontal of the mother satellite. Because $\dot{D} \le 0$ in ST and RE phases, the term $2\dot{D}\dot{\varphi}$ in Eq. (2) is zero or negative damping, which points out an instability of such motion. Therefore, a control algorithm, being proposed, should stabilize the system.

The objective of LRCA is to operate the tether reel mechanism so that the tether length rate $\dot{D}$ equals to $\dot{D}c$ defined as

$$\dot{D}c = \frac{k\dot{\upsilon} + k_1\dot{\varphi} - k_2\ddot{\upsilon}/\dot{\upsilon}}{\dot{\upsilon} + \dot{\varphi}}\dot{\upsilon}D, \qquad k \in (-0.75, 0.75), \qquad k_1 > 0$$

where k, k_1, and k_2 are choosable parameters of LRCA. With $k > 0$, $= 0$, or < 0, the subsatellite is operating in the DE, ST, or RE phase, respectively. A positive k_1 will stabilize all three phases of TSS motion. Suppose the tether reel mechanism is accurate enough so that

$$\dot{D} \approx \dot{D}C = \frac{k\dot{\upsilon} - k_1\dot{\varphi} + k_2\ddot{\upsilon}/\dot{\upsilon}}{\dot{\upsilon} + \dot{\varphi}}\dot{\upsilon}D \quad (3)$$

Therefore, the distance D is already determined by LRCA. With $\dot{D}$ defined in Eq. (3), Eq. (2) reduces to

$$\ddot{\varphi} + 2k_1\dot{\upsilon}\dot{\varphi} - 1.5(\mu/RM^3)\sin 2\varphi = -2k\dot{\upsilon}^2 - (1 - 2k_2)\ddot{\upsilon} \quad (4)$$

which determines the controlled motion of φ under LRCA. When $k_2 = 0.5$, Eq. (4) acquires a more simple form as

$$\ddot{\varphi} + 2k_1\dot{\upsilon}\dot{\varphi} - 1.5(\mu/RM^3)\sin 2\varphi = -2k\dot{\upsilon}^2$$

Periodic Motion in TSS

In the case of an elliptic orbit, the following expressions hold for the orbital elements:

$$\dot{\upsilon} = \sqrt{(\mu/p^3)}(1 + e\cos\upsilon)^2$$

$$\ddot{\upsilon} = -2(\mu/p^3)(1 + e\cos\upsilon)^3 e\sin\upsilon, \qquad RM = \frac{p}{1 + e\cos\upsilon}$$

For small eccentricity $e < 0.3$, $(1 + e\cos\upsilon)^{-1} \approx 1 - e\cos\upsilon$. With these expressions and replacing the independent variable t by υ, Eq. (4) acquires the following form:

$$\varphi'' + 2[k_1 - e\sin\upsilon(1 - e\cos\upsilon)]\varphi' - 1.5(1 - e\cos\upsilon)\sin 2\varphi = -2k + 2(1 - 2k_2)e\sin\upsilon(1 - e\cos\upsilon) \quad (5)$$

where $'$ means the derivative with respect to υ. The differential equation (5) is nonlinear and dissipative, and its coefficients are 2π-periodical so that it may have a periodic solution or periodic motion. There is a numerical iterative method of solving for the periodic motion as well as its stability.[9] First, let $X = (X1, X2)^T$, $X1 = \varphi$, and $X2 = \varphi'$, and transfer Eq. (5) into the state vector form:

$$X' = F(X, e, \upsilon)$$

$$F = \{X2, 1.5(1 - e\cos\upsilon)\sin 2X1 - 2[k_1 - e\sin\upsilon(1 - e\cos\upsilon)]X2 + 2(1 - 2k_2)e\sin\upsilon(1 - e\cos\upsilon) - 2k\}^T \quad (6)$$